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Research Paper

Analysis of Risk Measurement in Financial Companies

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Abstract

In this paper we extend the definition of risk measure from L^∞ to an arbitrary Polish space with special conditions. For this purpose we present a measure preserving transformation between two Polish spaces with special conditions.

Keywords: Polish Space; Risk Measure; Risk Management; Transformation

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1. Introduction

Risk management is a very important concept in financial mathematics and specially in a financial market.

For managing risk in a financial market we need to compute risk measure in a financial market which in [1, 2, 4, 5] is defined on L_∞. In this paper we extend the definition of risk measure from L_∞ to an arbitrary uncountable Polish space. For this purpose, we construct a measure preserving transformation between two Polish spaces which have special conditions.

2. Risk Measure

Risk measure is widely used as instrument to control risk. In fact, risk measures assign a real number to a risk in a financial market. As usual in actuarial sciences we assume that X describes a potential loss, but we allow X to assume negative values. Let (Ω, F, P) be a probability space and expectation of a random variable X with respect to P is denoted by $E[X]$.

2.1 Definition 1. Let X be the set of all functions $f : \Omega \to \mathrm{R}$. A mapping $\rho : X \to \mathrm{R}$ is called a risk measure if it has the following conditions [2, 3].

- Monotonicity: If $X \le Y$ then $\rho(X) \le \rho(Y)$;
- Translation invariance: if $m \in \mathrm{R}$, then $\rho(X + m) = \rho(X) + m$;
- Subadditivity: $\rho(X + Y) \le \rho(X) + \rho(Y)$;
- Positive homogeneity: if $\lambda > 0$, then $\rho(\lambda X) = \lambda\rho(X)$;
- Convexity: $\rho(\lambda X + (1-\lambda)Y) = \lambda\rho(X) + (1-\lambda)\rho(Y)$ for all $\lambda \in [0,1]$;
- Law invariance: If $P_X = P_Y$, then $\rho(X) = \rho(Y)$.

According to Artzner et al. a functional is called a coherent risk measure, if it is monotone, translation invariant, subadditive and positively homogeneous [2]. They show that any coherent risk measure has a representation.

$$\rho(X) = \sup_{Q \in \mathrm{Q}} E_Q(X), \qquad (1)$$

where Q is some set of probability measures. This means that $\rho(X)$ is the worst expected loss under Q, where Q varies over some set of probability measures. Follmer and Schied [6] introduced the weaker concept of ρ being a convex risk measure if it satisfies the condition of monotonicity, translation invariance and convexity. They show that any convex risk measure is of the form

$$\rho(X) = \sup_{Q \in \mathrm{Q}}(E_Q(X) - \alpha(Q)), \qquad (2)$$

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Where is a penalty function, which can be chosen to be convex and lower semi-continuous with $\alpha(Q) \geq -\rho(0)$.

2.2. *Definition* 2

Let (Ω,F,μ) be a probability space. Call for a partition P of Ω consisting of elements of F, $\sup_{I\in P} \mu(I)$ the norm of P, w.r.t. μ and denote it by $|P|_\mu$.

2.3. Definition 3

For a probability space (Ω,F,μ) a sequence $\{\triangle_n\}_{n\geq 1}$ of partitions of Ω is called a system of partitions if [7]:

1. for each $n \geq 1$, $\triangle_n$ is a countable collection of elements of F;
2. the collection $\cup_{n\geq 1}\triangle_n$ of subsets of Ω generates F;
3. $\lim_{n\to\infty} |\triangle_n|_\mu = 0$.

Call a system of partitions decreasing if for each $n \geq 1$, $\triangle_{n+1}$ is a refinement of $\triangle_n$. Henceforth $\triangle_n$, $n \geq 1$, denotes a system of partitions of Ω.

2.4. Definition 4

For $\omega \in \Omega$, $n \geq 1$, let $I_n(\omega)$ be the unique element of $\triangle_n$ containing ω. Call the sequence $I_n(\omega)$, $n \geq 1$, the $\omega-$ tower in the system.

2.5. Remark

Euclidean spaces and more generally, locally compact second countable Hausdorff topological spaces and hence complete separable, i.e. Polish, metric spaces, with Borel $\sigma-$ algebras and diffuse probability measures, when they admit such measures, yield decreasing systems of partitions which generate the Borel $\sigma-$ algebra.

3. Main results

In this section we extend the definition 2.1. For this purpose we present some theorems.

Let [0,1] be equipped by the Borel $\sigma-$ algebra **B** and the Lebesgue measure m. Let Ω be a Polish space and μ a non-atomic probability measure on F. Consider Ω and [0,1] equipped by the system of partition Δ and Δ', respectively.

3.1. Theorem 1

There is a transformation $X_b : \Omega \to [0,1]$ which has the following properties [7]:

1. Xb yields a natural one to one correspondence between the collection of towers of′; △ and △
2. Xb is F-B measurable and in fact $X_b^{-1}(B) = F$;
3. Xb transforms the measure μ on Ω to the Lebesgue measure m on I_0.

3.2. Theorem2

Let Ω_1 and Ω_2 be uncountable and Polish spaces. Then there is a measure preserving transformation between them.

3.3. Theorem 3

By above theorem, the definition of risk measure is extendable from L_∞ to an arbitrary uncountable Polish space.

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